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NAZ-003-001408 **Seat No.** ________

B. Sc. (Sem. IV) (CBCS) Examination

March / April - 2017

Mathematics : 401(A)

(Adv. Calculus & Linear Algebra) [New Course]

Faculty Code : 003

Subject Code : 001408

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following objectives : **20**

(1) $f(x,y) = \log\left(x^2 + y^2\right)$, find f_x and f_y.

(2) $w = \frac{y}{z} + \frac{x}{y} + \frac{z}{x}$ then $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + 2\frac{\partial w}{\partial z} =$ ______.

(3) Define Homogeneous function.

(4) If u is homogeneous function of x, y of degree n then

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \_\_\_\_\_.$$

(5) $u=\sqrt{x^3-xy^2}$ is homogeneous or not ? If yes then what about degree ?

(6) $x=r\cos\theta,\ y=r\sin\theta$ then $J\left(\frac{r,\theta}{x,y}\right)=$ ______.

(7) Define gradient of ϕ.

(8) $\bar{f}=\left(x^2y,-2xz,2yz\right)$ then $Curl\ \bar{f}$ at $(1,1,1)=$ _____.

(9) State Laplace Equation.

(10) Define Irrotational function.

(11) $\iint_R x^2+2y\ dxdy=$ ______ where $R=[0,1;0,2]$.

(12) $\iiint_R f(x,y,z)\,dx\,dy\,dz=\iiint f(r,\theta,z)$ _____ $dr\,d\theta\,dz$.

(13) $u=x+y$ and $v=x-y$ then $J\left(\frac{x,y}{u,v}\right)=$ _____.

(14) $\oint_C x\,dy-y\,dx=$ _____ where $C:x^2+y^2=1$.

(15) State the condition for $\int_C P\,dx+Q\,dy$ is path independent.

(16) Define Beta function.

(17) State the relation betwen Beta and Gamma function.

(18) State Cauchy Schwartz's inequality.

(19) For any inner product space $\|\overline{u}+\overline{v}\|^2+\|\overline{u}-\overline{v}\|^2=$ ____.

(20) $\overline{u}=(x_1, x_2)$, $\overline{v}=(y_1, y_2)$ and $\overline{u}\cdot\overline{v}=2x_1y_1+7x_2y_2$, then $\bullet$ is inner product on R^2 is true or false.

2 (a) Answer any three out of six : **6**

(1) If $x^3+y^3+z^3-3xyz=0$ then find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$.

(2) If $f(x,y)=\frac{1}{x^2}+\frac{1}{xy}+\frac{\log x-\log y}{x^2+y^2}$ then show that

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+2f(x,y)=0.$$

(3) Find Jacobian for cylindrical co-ordinates.

(4) $u=x^2-2y,\ v=x+y+z,\ w=x-2y+3z$ then find

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}.$$

(5) Prove that $div(\overline{r}\times\overline{a})=0$.

(6) $\phi(x,y,z)=x^2y+y^2z+z^2$ then find unit normal at $(1,1,1)$.

(b) Answer any three out of six : **9**

(1) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y};\ x \neq y$ then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

(2) If $u = f\left(y-z, z-x, x-y\right)$ then show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

(3) Expand $e^x \cos y$ in power of x and y upto three degree.

(4) Find maximum and minimum value of $f\left(x,y\right) = x^2 + 2y^2 - x$, which is possible.

(5) In usual notation prove that, $div\left(Curl\ \overline{f}\right) = 0$.

(6) If $\overline{f} = \left(2x+3y+az, bx+2y+3z, 2x+cy+3z\right)$ is irrotational then find a, b, c.

(c) Answer any two out of five : **10**

(1) State and prove "Euler's theorem" for homogeneous function of two variable.

(2) If $u=\phi(x,y)$ where $x=e^{a}\cos t$ and $y=e^{a}\sin t$ then show that

$$\left(\frac{\partial u}{\partial x}\right)^{2}+\left(\frac{\partial u}{\partial y}\right)^{2}=e^{-2a}\left[\left(\frac{\partial u}{\partial a}\right)^{2}+\left(\frac{\partial u}{\partial t}\right)^{2}\right].$$

(3) State and prove Taylor's theorem.

(4) If $\overline{f}$ and $\overline{g}$ are irrotational functions on D then show that $\overline{f}\times\overline{g}$ is a solenoidal function.

(5) Prove in usual notations $div\left(r^{n}\,\overline{r}\right)=(n+3)r^{n}$.

3 (a) Answer any three out of six : **6**

(1) Evaluate $\iint_{R} xy\,dx\,dy$ where R is the first quadrant of $x^{2}+y^{2}=a^{2}\cdot$

(2) Find $\int_{(0,0)}^{(2,4)} y\,dx+x\,dy$, where C is $y=x^{2}$.

(3) In usual notation prove that $\beta(p,q)=\beta(q,p)$.

(4) Prove that $\int_{0}^{2} x^{4}\left(8-x^{3}\right)^{-\frac{1}{3}}dx=\frac{16}{3}\beta\left(\frac{5}{3},\frac{2}{3}\right)$.

(5) In usual notation prove that,

$\beta(m,n) = \beta(m+1, n) \perp \beta(m, n+1)$.

(6) Determine $\overline{u} \cdot \overline{v} = u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$ is inner product on R^3 or not.

(b) Answer any three out of six : **9**

(1) Change the order of integration of $\int_0^1 \int_{1-y}^{1+y} f(x, y)\, dx\, dy$.

(2) Prove that $\oint_C \frac{y^2 dx - x^2 dy}{x^2 + y^2} = -\frac{8a}{3}$, where C is a circle with radius a.

(3) Prove that $\int_0^\infty \sqrt{y}\, e^{-y^2} dy \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy = \frac{\pi}{2\sqrt{2}}$.

(4) Prove that $\int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^6 \theta \, d\theta = \frac{8}{693}$.

(5) State and prove Triangle inequality.

(6) If u and v are vectors in real inner product space then show that $\|u+v\|^2 = \|u\|^2 + \|v\|^2$ if and only if $u \perp v$.

(c) Answer any two out of five : **10**

(1) Find $\int_{0}^{9} \int_{\sqrt{y}}^{3} \sin\left(\pi x^3\right) dx\, dy$.

(2) State and prove Green's theorem for a plane.

(3) State and prove relation between Beta and Gamma function.

(4) Prove that $\int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}} d\theta \int_{0}^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta = \pi$.

(5) Transform the basis $S = \{(1, 0, 0), (3, 7, -2), (0, 4, 1)\}$ into an orthonormal basis using Gram Schmidt process.